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SPACE TRAJECTORIES AND ERRORS IN TIME, FREQUENCY, AND TRACKING STATION LOCATION

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ABSTRACT

This paper demonstrates analytically how errors in tracking station time and frequency synchronization, as well as errors in station location, influence the accuracy of trajectory determination. Two systems, whose data are presently used for most of the more accurate orbit determination schemes, are described: a range and range rate system, and a radar system. All other systems are actually combinations of these two basic systems.

Rather simple analytical expressions are derived relating measuring errors with those of time and frequency synchronization, as well as ground tracking station location. For the range and range rate system, the error in range rate $\delta\dot{r}$ is used as a yardstick, and all other quantities are derived from it. For the radar system (not measuring range rate directly), the total local position vector, as measured by such a system is used as the yardstick to determine the necessary time synchronization and station location accuracy. Numerical examples are presented which are intended to show what accuracies of the mentioned quantities are needed for a good ground tracking system to make data most useful in determining accurate orbits and space trajectories.

CONTENTS

SPACE TRAJECTORIES AND ERRORS IN TIME, FREQUENCY, AND TRACKING STATION LOCATION*

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SUMMARY

General

Frequency—or more definitively, phase change over a period of time—is among the few physical quantities that can be measured with extreme accuracy. If, therefore, such measurements can be used to determine orbits, one would expect the determinations of such orbits to be very accurate. This is in fact the case, and Doppler-frequency measurements are so used on an almost daily basis, particularly for computing trajectories in deep space. In brief, the Doppler frequency component in an electromagnetic signal from a spacecraft is often used as a yardstick.

One could use, even more appropriately, an error quantity at a particular target: for instance, the closest approach to the moon or a planet should be, say, 100 meters or 10,000 meters. However, using an error quantity in this manner may necessitate system requirements that cannot be met because of physical limitations. These limitations include systems noise, bias errors, and tropospheric and ionospheric propagation that will disturb the transmitted and received electromagnetic signals.

For this is why it has been decided here to limit the range rate. In any event, the final results should be almost the same. Since this yardstick is an extremely accurate one ($\Delta\nu/\nu \approx 10^{-10}$ to 10^{-12} per day), all other quantities involved must be appropriately accurate. For example, for the Doppler measurement to be exploited to its fullest, the sender frequency must be very accurate, and the time synchronization between tracking stations, as well as their location on the surface of the earth, must be determined to a high degree of certainty. This paper outlines how much and in what ways these quantities and their errors are approximately related to each other. Coherent two-or three-way, phase-locked Doppler systems as well as radar systems are also discussed.

Establishing reasonable design limits cannot be mathematically rigorous. Human experience and judgment are important factors. Nevertheless, it is felt that the numerical values arrived at will give the reader a good picture of the problem.

*Published also as NASA Technical Memorandum X-55798, 1966.

Relative-Frequency Stability

As can be noted later on (from Equation 9) one needs a relative frequency stability of

$$\left(\frac{\delta\nu_{D_2}}{\nu_{D_2}}\right)_{tm} \doteq \left(\frac{\delta\nu_0}{\nu_0}\right)_{2\tau} < \frac{\delta\dot{r}}{\dot{r}} \doteq 2\times 10^{-9} ,$$

where the first term refers to the measured Doppler shift during the measuring time t_m, the second term refers to the relative stability of the local standard oscillator over the time the signal travels from the station to the spacecraft and back, $\delta\dot{r} = 0.01$ cm/sec (the assumed yardstick), and $\dot{r}$ is assumed to equal 50 km/sec for an average maximum interplanetary range rate. In addition, the value $(\delta c/c)$ must be considered as an unknown in the orbit determination process in order that the above values of $\delta\dot{r}$, etc., can be used.

Station Time Synchronization

Equations 12 and 13 (see further on) give $\delta t = 0.1$ to 0.2 msec for the earth-moon space using $\delta\dot{r} = 0.1$ cm/sec and full earth acceleration $\ddot{\rho} \doteq 1000$ cm/sec^2 (a pessimistic value proper for near the earth).

Similar $\delta t \doteq 0.2$ msec is the needed time synchronization error for a radar depending on the assumed angular bias error of the radar considered. Equations 16, 17, and 17a are applicable for this case.

For interplanetary missions where $\ddot{\rho}$ is small (in this case $\ddot{\rho} \doteq 0.6$ cm/sec^2 is the accelera-
tion due to the sun for 1 AU), even with $\delta\dot{r} = 0.01$ cm/sec (compared to 0.1 cm/sec for near-earth
tion due to the sun for 1 AU), even with $\delta\dot{r} = 0.01$ cm/sec (compared to 0.1 cm/sec for lunar space), a value of $\delta t \doteq 2$ msec is adequate as given by Equation 13. Only in the special case of a planetary approach or a planetary fly-by, timing errors of $\delta t \doteq 10$ to 20μ sec seem to be the orbit for time synchronization, can fulfill this requirement without additional equipment or methods.

Station Location Errors

To make full use of a Doppler error as small as 0.01 cm/sec requires a station location accuracy of:

$$|\delta\vec{R}| \doteq 1.5 \text{ to } 5.5 \text{ meters} ,$$

depending on the use of Equation 26 and 27. This range of accuracy is something to aim at.

In the case of a radar system, a system error range of

$$|\delta\vec{R}| \doteq 3.5 \text{ to } 11.0 \text{ meters}$$

will be necessary for future systems according to Equation 30 and its associated assumptions.

Frequency Synchronization

For a *two-way* Doppler system, similar values hold—as stated under Relative-Frequency Stability above. In this case one speaks not of frequency synchronization as such but, rather, of frequency stability during the travel time of the electromagnetic signal. (This is, of course, different for the *three-way* Doppler system, as applied to lunar and planetary missions.)

Equation 19, shows that to make full use of the Doppler ($\delta\dot{r} = 0.01$ cm/sec) requires a frequency synchronization between stations 1 and 2 of

$$\frac{\delta\nu_{12}}{\delta\nu_0} \lessapprox \frac{2}{3} 10^{-12} .$$

Or, in brief, field-worthy hydrogen masers must be developed and used.

Note that the errors considered—namely, the errors in frequency synchronization, tracking station time synchronization, and location of the tracking stations on earth—are to be considered as *bias* errors. This is obvious, since all these quantities stay approximately constant while tracking measurements are taken and the orbits are determined.

FREQUENCY STABILITY

One of the first questions to be answered in designing a coherent Doppler tracking system is: what frequency stability of the transmitter is required?

Figure 1 shows the principle (References 1 through 5) used to detect the Doppler frequency shift ν_{D_2} for a two-way Doppler system.

The multiplied frequency ν_0 of a stable source is transmitted to the spacecraft, received there, translated to ensure no interference, retransmitted to the same ground receiving system together with the translation factor k, and finally fed into a mixer (Reference 5). Here, the difference frequency between the

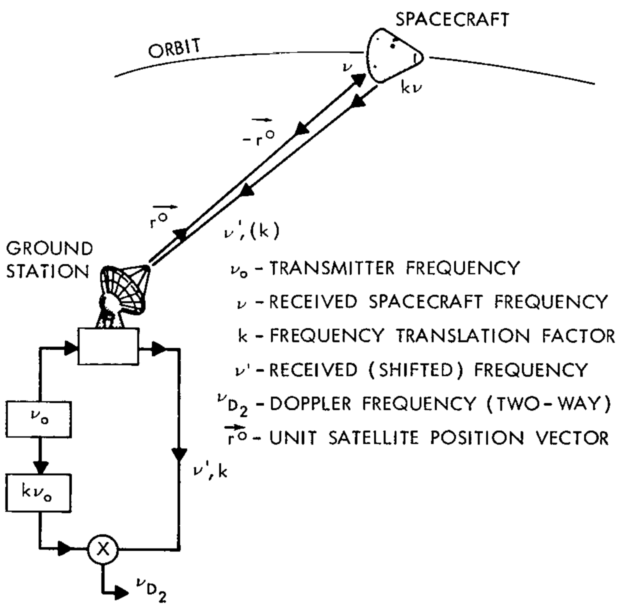


Figure 1—Two-way Doppler system.

signal sent to the spacecraft and that received back on the ground is extracted as the Doppler frequency ν_{D_2}; this is trustworthy, provided that the oscillator maintains constant frequency during the signal travel time, 2τ. The significance of "constant" will be shown later.

The Doppler Shift

The special relativistic equation for the Doppler shift reads (References 1 through 4):

$$\nu = \nu_0 \frac{1 - \frac{1}{c}(\vec{v} \cdot \vec{r}^{\,\circ})}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} , \tag{1}$$

where $\vec{v}$ is the relative velocity between spacecraft and receiving station ($v = |\vec{v}|$); c the speed (scalar) of light; ν the frequency at the spacecraft; ν_0 the transmitted frequency, and $\vec{r}^{\,\circ}$ the unit position vector from the station to the spacecraft in the moving frame of reference. Further,

$$\dot{r} = (\vec{v} \cdot \vec{r}^{\,\circ}) \tag{2}$$

is the spacecraft range rate with respect to the tracking station. A similar equation holds for the return path, as shown in References 1 through 4. The ratio of the received frequency ν^1 and the transmitted frequency ν_0 is shown to equal

$$\frac{\nu^1}{\nu_0} = \frac{1 - \frac{1}{c}\dot{r}}{1 + \frac{1}{c}\dot{r}} . \tag{3}$$

The Doppler frequency for the so-called two-way mode ν_{D_2} is known simply as:

$$\nu_{D_2} = \left(\nu^1 - \nu_0\right) . \tag{4}$$

The factor k can be assumed to be 1 since this value is compensated for on the ground, as shown in Reference 5. As mentioned, the frequency translation (k) was performed only to eliminate interference at the spacecraft.

Equations 3 and 4 give for the range rate

$$\frac{dr}{dt} = \dot{r} = \frac{-c}{2}\frac{\nu_D}{\nu_0} \cdot \frac{1}{1 + \left(\nu_D/2\nu_0\right)} \tag{5}$$

Equation 5 relates Doppler frequency ν_D, transmitter frequency ν_0, speed of light c, range rate $\dot{r}$ between spacecraft and transmitting stations in exact form as given by the special theory of relativity. (No gravitational terms are considered since their effects are relatively small.)

The Relative Errors

Equation 5 will now be used to answer the question of frequency stability using $(d\dot{r}/\dot{r})$* as a "yardstick." Since only small variations are to be considered and $\nu_D/\nu_0 \lll 1$, Equation 5 can be simplified for the following variational analysis, that is:

$$\dot{r} \doteq \frac{-c}{2} \frac{\nu_D^{**}}{\nu_0} . \tag{6}$$

Varying Equation 6 and adding the results in the Gaussian sense leads to

$$\left(\frac{\delta \dot{r}}{\dot{r}}\right)^2 = \left(\frac{\delta c}{c}\right)^2 + \left(\frac{\delta \nu_D}{\nu_D}\right)^2 + \left(\frac{\delta \nu_0}{\nu_0}\right)^2 . \tag{7}$$

This equation shows that the relative error $\delta\dot{r}/\dot{r}$ in the determination of $\dot{r}$ depends on the relative error in the speed of light $\delta c/c$, the relative error in the Doppler frequency $\delta\nu_D/\nu_D$, and the transmitter frequency $\delta\nu_0/\nu_0$. Decreasing $\delta\dot{r}$ or $\delta\dot{r}/\dot{r}$ means reducing all the other three quantities.

First, it is necessary to improve the relative error $(\delta c/c)$ of the speed of light (at present $(\delta c/c) = 3 \times 10^{-7}$ (References 6 through 9), then measure the relative Doppler frequency $(\delta\nu_D/\nu_D)$ to at least the same accuracy. This means that: a certain measuring time t_m, and measuring time error δt_m must be secured to make a frequency measurement to a given accuracy (Reference 10), that is:

$$\frac{\delta \nu_D}{\nu_D} \doteq \frac{\nu_D}{\nu_0} \cdot \frac{\delta t_m}{t_m} \doteq \frac{2\dot{r}}{c} \cdot \frac{\delta t_m}{t_m} , \tag{8}$$

assuming that the quantities contributing to noise (noise modulation index, signal to noise ratio) are small enough compared with $(\delta t_m/t_m)$, which is true for the systems and methods considered here.

For example, a carrier frequency $\nu_0 = 2000$ Mc, a measuring time $t_m = 1$ sec, and an error in the measuring time $\delta t_m = 10^{-6}$ sec give a value of

$$\frac{\delta \nu_D}{\nu_D} = \frac{2}{3} \cdot 10^{-4} \cdot 10^{-6} \cdot = \frac{2}{3} 10^{-10} \cdot (\text{for } \dot{r} = 10 \text{ km/s}) .$$

In addition, of course, the transmitter standard oscillator must be "constant" during the travel time 2τ of the electromagnetic wave from the transmitter to the spacecraft. A shift during this

*During the course of this paper normalized errors such as δ_c/c, $\delta_{\dot{r}}/\dot{r}$. . . . etc., are often used for simplicity of the mathematical expressions.

**Sign of Doppler frequency ν_D is neglected from now on since a "negative," frequency has no real meaning.

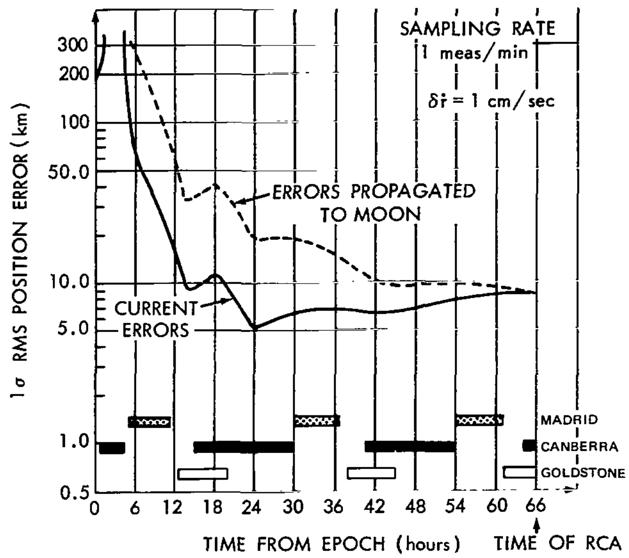


Figure 2—Position errors of a lunar transfer trajectory.

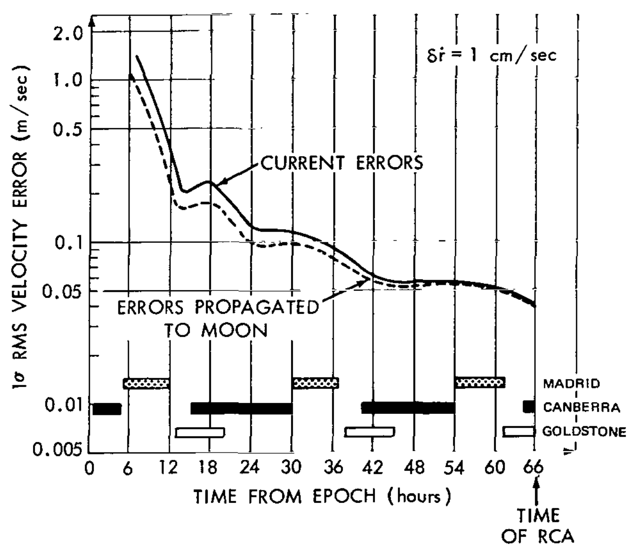


Figure 3—Velocity error of a lunar transfer trajectory.

time cannot be detected by the mixing process shown in Figure 1 and described in Reference 5. In brief, one needs:

$$\frac{\delta c}{c} \doteq \left(\frac{\delta \nu_D}{\nu_D}\right)_{tm} \doteq \left(\frac{\delta \nu_0}{\nu_0}\right)_{2\tau} . \qquad (7a)$$

Assume, as an example, that all three quantities equal 10^{-6} (a conservative value) than $\delta \dot{r} = \sqrt{3}.10^{-6}\,\dot{r}$. For flights between the earth and the moon, $\dot{r}_{ar} = 3$ km/sec $= 3.10^5$ cm/sec and thus $\delta \dot{r} = 1.0$ cm/sec which is at present adequate for purposes of trajectory determination for the Apollo project (References 7, 8, and 11 through 16). (See also Figures 2 and 3.) Figure 2 depicts the instantaneous position error (propagated to the moon) of a lunar transfer using a bias range-rate error of 1 cm/sec plus a 1 cm/sec noise component superimposed. Station errors as given in References 7 and 8 are also included to show realistic values as used for these Apollo Navigation studies. Figure 3 shows the same for the spacecraft velocity.

For interplanetary flights one must decrease the errors in range rate, in order to get a more reasonable trajectory. Figures 4 and 5 show the position and velocity errors of a Jupiter probe as an example when tracked by a two-way Doppler system using three ground stations approximately 120 degrees separated in longitude. The errors of twenty days of tracking are shown; these errors are then propagated to 260, 458, and 500 days respectively (planets intercept) along the trajectory. This is in order to be able to find if it is reasonable to make a midcourse maneuver after, say, 20 days of tracking based upon the range-rate accuracy requirements discussed here.

Since at present both quantities—$(\delta \nu_D/\nu_D)$ and $(\delta \nu_0/\nu_0)$—can be determined far better than $(\delta c/c)$ one must improve on the latter to improve $(\delta \dot{r}/\dot{r})$ as shown by Equations 7 and 7a. This should possibly be done by an orbit-independent method in a laboratory, since c and thus $\delta c/c$ can be calculated (considered as an additional unknown in the orbit computation) together with the orbit parameters station locations, etc. The problem involved here is that all our constants are based upon the velocity of light within 3×10^{-7} as quoted before (Reference 17). The velocity of light is a primary physical constant and occupies a key position in astronomy (References 17 and 18). The velocity of light is involved via the "light time" (time the light needs to transverse 1 AU, the heliocentric distance of the center of mass of the earth + moon system) with the solar parallax, the mean earth radius, the eccentricity of the earth orbit, the Gaussian gravitational constant, etc. This means that all these quantities must be changed accordingly to adjust our physical constants in the universe in terms of distance, distance variation, time, etc. (Reference 17).

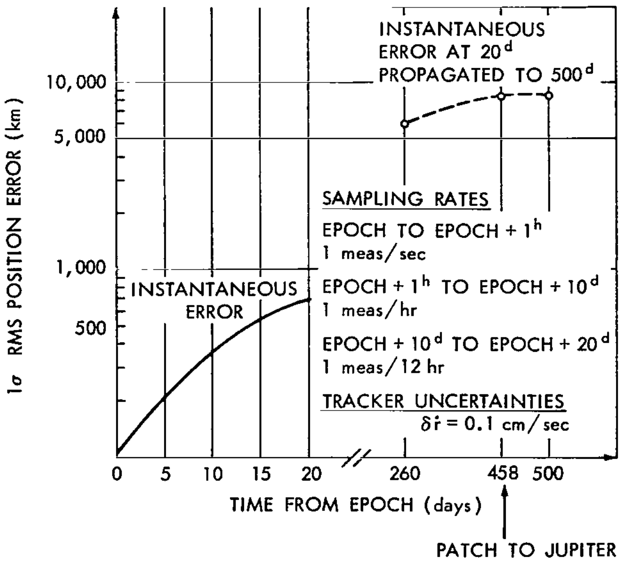


Figure 4—Position errors of a Jupiter trajectory.

For interplanetary flights (References 19 through 21), values of $\delta \dot r$ = 0.02 cm/sec have been obtained over 1-minute of measuring time. Thus, using 0.01 cm/sec as a standard, one would need for $\dot r = 5.10^6$ cm/sec the following accuracies (extreme values):

$$\frac{\delta \dot r}{\dot r} \doteq \left(\frac{\delta c}{c}\right) \doteq \left(\frac{\delta v_D}{\nu_D}\right)_{tm} \doteq \left(\frac{\delta v_0}{v_0}\right)_{2\tau} \lesssim 2.10^{-9} \quad . \quad (9)$$

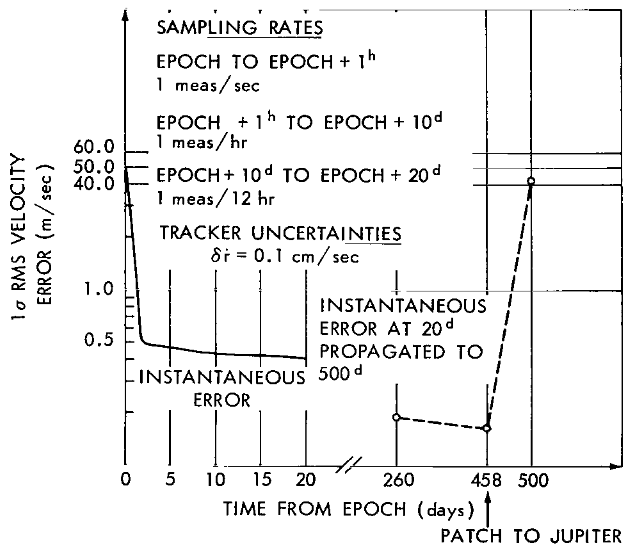


Figure 5—Velocity errors of a Jupiter trajectory.

On the assumption of a good signal-to-noise ratio of 10 db or more, and Doppler measuring times on the order of minutes (Reference 10), atmospheric disturbances seem to be the only limiting factors (References 19 through 22). As shown in Reference 22, this limit is of the order of 0.01 cm/sec rms when elevation angles $\epsilon \gtrsim 10$ degrees are used and daily tropospheric correction terms are applied in the order of $\delta N = \pm 5$ where $N = (n - 1) \cdot 10^{-6}$, and n is the index of refraction. The only unknown quantity is $(\delta c/c)$ to the accuracy stated, thus a better determination of $\delta c/c = 3.10^{-7}$ as shown in Reference 6 is needed. Laboratory measurements of the velocity of light will probably not reach an accuracy of 1p 10^{10} or better in the near future. One way to circumvent the errors of the known value of c is to introduce this quantity as an "unknown" into orbit determination (Jet Propulsion Laboratory).

STATION TIME SYNCHRONIZATION

One of the next logical questions to answer is: To what accuracy should the time be synchronized between tracking stations used for orbit determination?

A relationship between the time error δt and the tracking quantities to be measured will be derived in the following. A CW range-rate system and a radar system are considered since their time synchronization requirements differ according to their different "yardsticks." (A radar does not "measure" $\dot{r}$ as such, for instance.)

Synchronization Between Station Time and Orbit Time

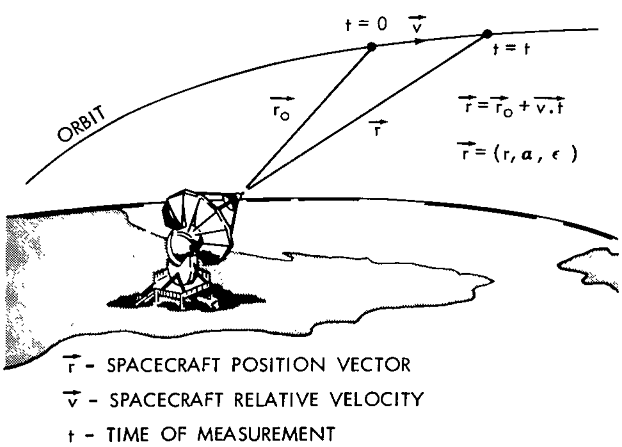
For orbit determination, the local or station measurements r, $\dot{r}$, etc. (or any other measurement) are made at a certain time t. These values are then transmitted from the station to the computing center and there used for orbit determination. Thus, the station clock must be synchronized to the computing center "clock" to within a certain limit, say δt. What this value should be in order to make full use of our standard $\delta\dot{r}$ is shown in the following.

Considering only small deviations within a CW range-rate system, one can write to a first-order accuracy for the variation $\dot{r}$ the following:

$$\delta\dot{r} \doteq \ddot{r}\,\delta t \;, \tag{10}$$

since

$$\ddot{r} = \frac{\delta}{\delta t}\dot{r} \;,$$



$\vec{r}$ - SPACECRAFT POSITION VECTOR
$\vec{v}$ - SPACECRAFT RELATIVE VELOCITY
t - TIME OF MEASUREMENT
$\vec{r}_o$ - PROPER POSITION VECTOR FOR SAY t = 0 AND NO SYNCHRONIZATION ERROR

Figure 6—Radar measurement of slant range vector.

where $\ddot{r}$ is the time derivative of the range rate $\dot{r}$ and δt is the time error to be determined here.

The spacecraft inertial position vector (Figure 6) $\vec{\rho}$ is

$$\vec{\rho} = \vec{R} + \vec{r} \tag{11}$$

and acceleration are:

$$\ddot{\vec{\rho}} = \ddot{\vec{R}} + \ddot{\vec{r}} \tag{12}$$

Borrowing terms from vector analysis, that is, $\ddot{\vec{R}} = (\vec{\omega}\times\vec{R})$ and further $\ddot{\vec{R}} = \vec{\omega}\times(\vec{\omega}\times\vec{R})$, one obtains

$$\ddot{\vec{\rho}} = \left[\vec{\omega}\times(\vec{\omega}\times\vec{R})\right] + \ddot{\vec{r}} \tag{12a}$$

Using Equations 12a and 10 one obtains approximately:

$$\delta t = \frac{\delta \dot{r}}{|\ddot{\vec{\rho}} - \ddot{\vec{R}}|} . \tag{13}$$

Here one has to remember two points, namely: the range rate error $\delta \dot{r}$ was previously assumed to be 0.01 cm/sec, on the basis of deep-space mission results, and the range rate is *not* the only measurable quantity used for orbit determination for near-earth satellites except for the TRANET spacecraft. From experimental data such as Goddard spacecraft as well as lunar orbiter evaluation, a value of $\delta \dot{r} \lesseqqgtr 1$ cm/sec emerges. Improvements are under way, so that an assumption of $d\dot{r} = 0.1$ cm/sec for earth and lunar space satellites seems reasonable for the near future. Using this value, namely $d\dot{r} = 0.1$ cm/sec and $\ddot{\rho} = 10^3$ cm/sec^2 as the near-earth and lunar-space yardstick, one obtains, using Equation 13,

$$\delta t = \frac{0.1}{10^3} = 10^{-4} \text{ sec} = 100 \ \mu\text{sec} . \tag{13a}$$

The value $|\ddot{\vec{R}}| = 3$ cm/sec^2 has been neglected, as compared with $\ddot{\rho} \doteq 1$ g $\doteq 1000$ cm/sec^2.

For interplanetary travel, where the acceleration terms are small (in the order of a few cm/sec^2), timing errors can be appropriately large as dictated by Equation 13.

The value given in Equation 13a should then suffice for most space tracking problems.

The only time where more stringent requirements are to be considered is during a planetary approach. For such cases, the value given in Equation 13a may have to be eventually reduced to say 10 to 20 μsec ($\ddot{\rho} \gtreqless 1000$ cm/sec^2, $\delta \dot{r} = 0.01$ cm/sec). Here, on the other hand, single tracking stations are mostly involved for approximately 8 hours a day, if necessary. The station time synchronization method suggested by JPL (Reference 27) can be applied, reducing the timing errors to the above-mentioned values without elaborate additional station equipment and/or other methods (e.g., VLF).

A radar system for near-earth orbit calculations needs more than $\dot{r}$ information. Angular data (azimuth, elevation, or equivalent) and range data (radar) are used to determine orbits for manned flight programs (References 7, 8, 11, and 12). For these programs, $\dot{r}$ and $\delta \dot{r}$ will not serve as yardsticks; and yet time errors do enter the orbit analysis. (Their effect here is not severe, as will be shown.)

Suppose (to assume the worst) that a near-earth spacecraft moves at 10 km/sec (during a lunar transfer orbit) and that the radar system would "see" a velocity of almost the full 10 km/sec. What would the time accuracy have to be to make the position measurement ($\vec{r}$, a vector quantity) accurate to a certain predescribed number of meters (for example, 5 to 10)?

In this case, one can write (see Figure 6)

$$\vec{r} = \vec{r}_0 + \vec{v}t \, , \tag{14}$$

where $\vec{r}$ is the measured slant range vector $\vec{r}$ at $t = t$ (range r, azimuth α, elevation ϵ) $\vec{r}_0$ is the initial position vector at $t = 0$, and $\vec{v}$ is the velocity of the spacecraft relative to the radar system. If the stations are synchronized completely, the value $\vec{r} = \vec{r}$ at $t = 0$. This of course, can never be achieved. Therefore, varying Equation 14 yields

$$\delta\vec{r} = \delta\vec{r}_0 + \delta\vec{v}t + \vec{v}\,\delta t \, . \tag{15}$$

Considering all these errors one obtains, in the Gaussian sense, rearranging the terms and introducing r, α, and ϵ:

$$v^2 \delta t^2 = \delta r^2 + r^2 \left(\delta\epsilon^2 + \delta\alpha^2\right) + \delta r_0^2 + r_0^2 \left(\delta\epsilon_0^2 + \delta\alpha_0^2\right) + \delta v^2 t^2$$

or

$$\delta t = \frac{1}{v} \sqrt{2\delta r^2 + 2r^2\left(\delta\epsilon^2 + \delta\alpha^2\right) + \delta v^2 t^2} \, . \tag{16}$$

Since δr^2 and $\delta\nu^2 t^2$ are always small compared to the middle term, (t probably less than a millisecond), one can write

$$\delta t \doteq 2 \frac{r}{v} \delta\measuredangle_B \, , \tag{17}$$

where $\delta\measuredangle_B$ is the bias error in elevation and azimuth assumed to be equal for reasons of simplicity. Equation 17 thus gives an estimate of the time synchronization needed for a good radar system. For example, assume a near-earth orbit with $v \doteq 8000$ m/sec, a radar slant range $r = 200$ km, and a elevation and azimuth bias $\delta\measuredangle_B = 0.02$ mrad; then $\delta t = 0.5$ msec.

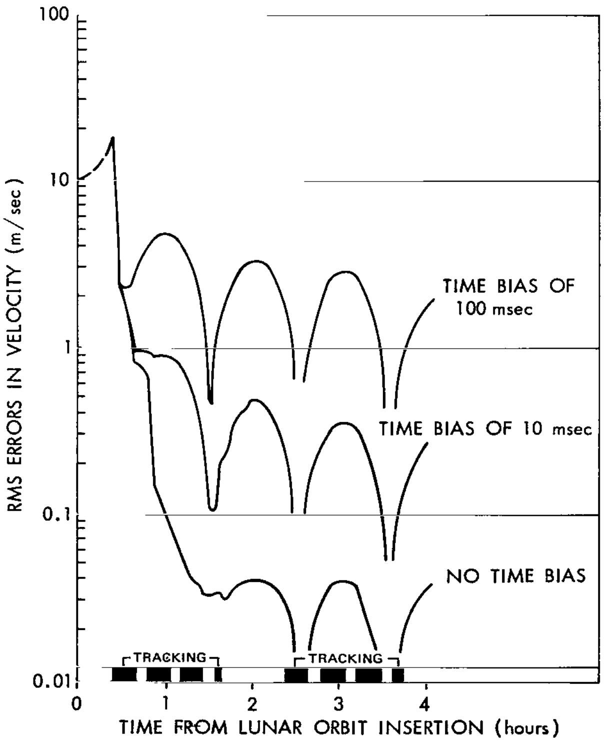
For near-earth orbit determination, the angular measurements of a radar system play a major role, especially when only short tracking times on the order of 60 to 100 sec are available. In this case, one obtains

$$\delta t \doteq \frac{\delta\epsilon_B}{\dot{\epsilon}_{max}} \doteq \delta\epsilon_B \frac{h}{v} \, , \tag{17a}$$

where $\delta\epsilon_B$ is the bias error in the random elevation angle, v is the orbital speed of the spacecraft, h is its height, and $\dot{\epsilon}_{max}$ is the maximum angular acceleration. Equation 17a represents a worst-case overhead pass. Using again as an example $v = 8000$ m/sec, $h = 200$ km, $\delta\epsilon_B = 0.02$ mrad, gives $\delta t = 0.25$ msec. Thus even for sophisticated, and properly calibrated radars, a true synchronization of 0.2 msec is adequate, since range, azimuth, and elevation measurements play a role in the needed time synchronization, as Equation 17 indicates. The angular errors push the needed time synchronization to a larger value.

In Reference 26 it is shown how an error in station time synchronization influences the errors in the final Apollo orbits obtained. In general, under present assumptions, Apollo systems (References 7 and 8), timing errors of approximately 10 msec do not appreciably influence the earth-parking and lunar transfer orbits. This is different for lunar orbits, where 10 msec do make the error ten times as great as with 1 msec. See Figure 7 (taken from Reference 26).

A simple and elegant method for time synchronization was suggested in a JPL report (Reference 27). The range is measured from two stations using a CW ranging system. From one of the ranges the "time" of one of the stations can be determined with respect to the other. The error in time synchronization $\delta t = (1/c)\,\delta\,pos$, where $\delta\,pos$ is the position error of the spacecraft and c is the speed of light. Even a large position error, say 3 km (see Figure 5), results in $\delta t = (1/3) \times 10^{-8} \times 3.10^3 = 10^{-5} = 10\,\mu$sec, a rather small time-synchronization error.



ORBITAL PARAMETERS

EQUATOR OF DATE COORDINATES - MOON - CENTERED
T = SEPT 12, 1969 77.170049
X_1 = 165.63936 nm $\dot{X}_1$ = -5208.365 ft/sec
X_2 = -919.37695 nm $\dot{X}_2$ = -817.04316 ft/sec
X_3 = -415.86151 nm X_3 = - 268.21399 ft/sec

INSERTION ERRORS
(1σ)
POSITION = 5196 m MEASUREMENT BIAS UPDATED
VELOCITY = 10.4 m/sec STATION LOCATION UNCERTAINTY NOT UPDATED
TIME BIAS NOT UPDATED

HORIZON

ELEVATION ANGLE ≥5°

STATION	SYSTEM	STATION LOCATION UNCERTAINTIES			MEASUREMENTS	SAMPLING RATE	MEASUREMENT UNCERTAINTIES	
		NORTH (1σ) (m)	EAST (1σ) (m)	UP (1σ) (m)			NOISE (1σ) (m/s)	A-PRIOR BIAS (1σ) (m/s)
CNB	USBS	63.0	60.0	61.0	R	1 PER MIN	0.00061	0.0091
CRO	USBS	60.0	64.0	60.0	R	1 PER MIN	0.00131	0.0610
HAW	USBS	53.0	74.0	73.0	R	1 PER MIN	0.00131	0.0610

Figure 7—RMS velocity errors during lunar orbital phase (measurement biases updated).

FREQUENCY SYNCHRONIZATION

As outlined under Frequency Stability, the frequency of a transmitting system must be constant only to a specified value during the travel time of the signal.

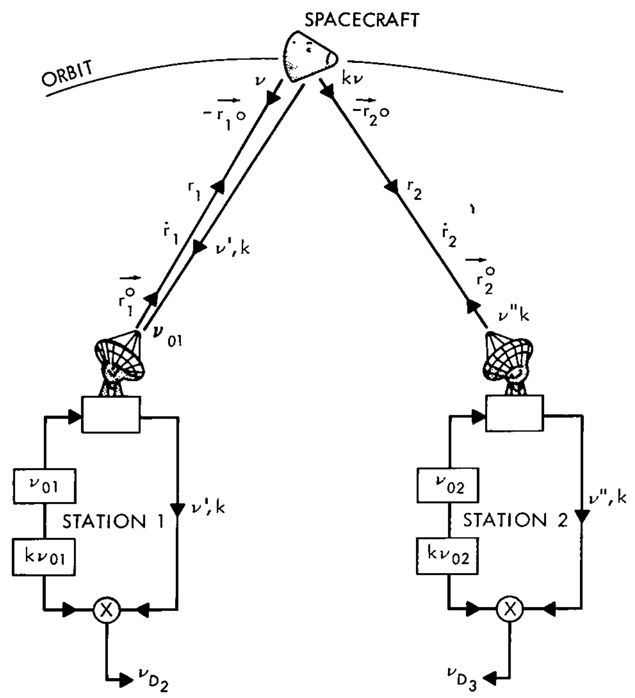


Figure 8—Three-way Doppler system.

This is not true, of course, when the transmitted signal is received at a "secondary" station as well as at the transmitting site. This technique, the three-way Doppler (a "pseudo Doppler") is applied for orbit determination of the lunar orbiter and will be used for the lunar-orbit determination during the Apollo mission (References 7, 8, and 11. See also Figure 8.)

Using exactly the same approach as outlined under Frequency Stability, one can write an equation similar to Equation 9, but

$$\frac{\delta\nu_{D3}}{\nu_{D3}} \leq \frac{\delta\nu_{12}}{\nu_{01}} \doteq \frac{\delta\nu_{12}}{\nu_{02}} < \frac{\delta\dot{r}_{ps}}{\dot{r}_{ps}} \doteq 2 \times 10^{-9} , \quad (18)$$

where the pseudo Doppler frequency (References 7 and 8) at the second station is ν_{D3} (three-way Doppler) and $\delta\nu_{12} \equiv (\nu_{01} - \nu_{02})$ is the difference frequency (error in frequency synchronization) between stations 1 and 2 (see Figure 7) and $\dot{r}_{ps}$ is the pseudo range rate between the two stations. Equation 9 must be modified, since $\delta\nu_{12}$ (unknown) will be included in the three-way Doppler as can be seen by inspecting Figure 7. From Equation 18 one obtains for $\delta\nu_{12}$ a value similar to $\delta\nu_{D3}$. Obviously when ν_{D2} is measured, the error $\delta\nu_{12}$ is contained in the error of ν_{D2}, namely $\delta\nu_{D3}$. Equation 18 can be written as

$$\delta\nu_{12} \doteq 2\nu_0 \frac{\delta\dot{r}_{ps}}{c} , \quad (19)$$

where $\dot{r}_{ps}$ represent the three-way (pseudo) Doppler. Example:

$$\delta\dot{r}_{ps} = 0.1 \text{ cm/sec}$$

$$\nu_0 = 2 \times 10^9 \text{ cps} .$$

then

$$\frac{\delta\nu_{12}}{\nu_0} = \frac{2}{3} \cdot 10^{-11}$$

is an extremely small value.

This number is obtainable with hydrogen masers (References 23 through 25). Thus a three-way Doppler system (References 7 and 8) seems to be one of the first practical systems requiring a frequency standard of hydrogen-maser quality. A frequency difference $\delta\nu_{12}$ between these two standards still results in a bias error in $\dot{r}_{ps}$, which is more damaging for the orbit analysis (less accurate) than a random error. A bias error is the same, no matter how many measurements are made; a random error decreases with $N^{-1/2}$, the number of measurements (References 13 through 15).

STATION LOCATION ERRORS

Possible location errors, compatible with a range rate system and the "yardstick $\delta\dot{r}$," as well as with a radar system, will now be discussed. These errors are considered a limit for these systems, based on their accuracy assumptions.

For a CW Range-Rate System

The aim here is to relate a station deviation $\delta\vec{R}$ (the variation of the station position vector $\vec{R}$) to the range rate deviation $\delta\dot{r}$.

Range rate $\dot{r}$, a scalar quantity, is simple; the projection of the spacecraft velocity $\vec{v}$ minus the station velocity $\dot{\vec{R}}$ onto the unit local satellite position vector $\vec{r}^{\circ}$ (see Figure 9). In vector notation, this means

$$\dot{r} = \left(\vec{r}^{\circ} \cdot (\vec{v} - \dot{\vec{R}})\right) = \frac{1}{r}\left(\vec{r} \cdot (\vec{v} - \dot{\vec{R}})\right) . \qquad (20)$$

Varying Equation 20 yields the deviation $\delta\dot{r}$ as a function of the station position error $|\delta\vec{R}|$.

Before this is done, Equation 20 will be regrouped for ease of calculations and the value

$$\frac{d}{dt}\vec{R} = \dot{\vec{R}} = (\vec{\omega} \times \vec{R}) \qquad (21)$$

borrowed from basic vector analysis introduced. The quantity $\vec{\omega}$ is the rotation vector of the earth ($|\vec{\omega}| \doteq 7.3\ 10^{-5}\ \sec^{-1}$).

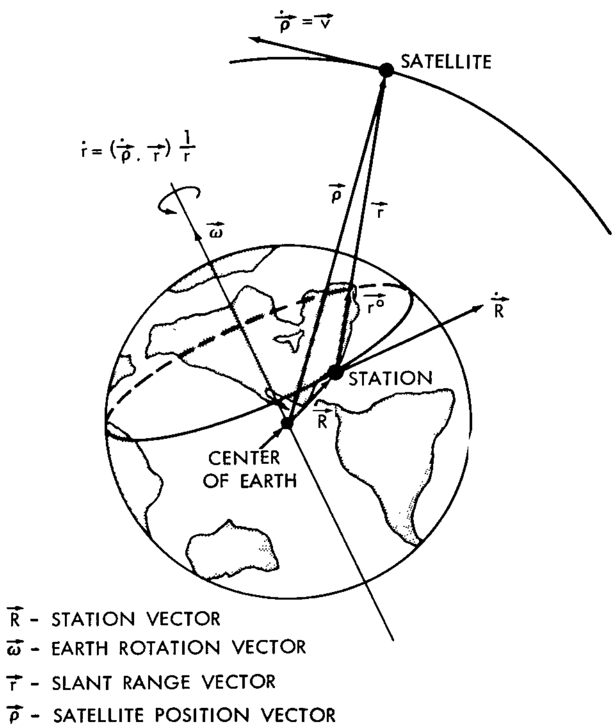


Figure 9—Tracking station-satellite geometry.

Equation 20 can now be written as

$$r\dot{r} = \left[(\vec{r} \cdot \vec{v}) - (\vec{r} \cdot (\vec{\omega} \times \vec{R}))\right] \tag{23}$$

and varying yields after some manipulation

$$r\delta\dot{r} = \left(\delta\vec{R} \cdot \left(\vec{v} - (\vec{\omega} \times \vec{R})\right) - \dot{r}\vec{r}^{\circ} + (\vec{r} \times \vec{\omega})\right) \tag{24}$$

with the assumption of $\delta\vec{v} = 0$ and $\delta\vec{\omega} = 0$. In order to "see" the influence of the station location error $\delta\vec{R}$ onto the range rate $\dot{r}$, one has to assume a perfect orbit, that is $\delta\vec{v} = 0$. In addition, the variation $\delta\vec{\omega}$ of the earth rotation is certainly zero during the short time considered.

Regrouping Equation 24 yields

$$\delta\dot{r} = \left(\delta\vec{R} \cdot \left(\frac{1}{r}\vec{u} + (\vec{r}^{\circ} \times \vec{\omega})\right)\right) \tag{25}$$

or

$$\delta\dot{r} = \frac{1}{r} u\delta R \cos \measuredangle_1 + \omega\delta R \cdot \sin\phi \cos \measuredangle_2 \tag{26}$$

Since the angles $\measuredangle_1$, $\measuredangle_2$ are not known, their statistical average value

$$\left[\frac{1}{2\pi}\int_{2\pi} \cos^2 x \, dx\right]^{1/2} = \frac{1}{\sqrt{2}}$$

is used. For the velocity $|\vec{u}| = |\vec{v} - \vec{r}^{\circ}\dot{r}|$ the value $(v^2 - \dot{r}^2)$ is used as an approximation. Thus, Equation 26 finally reads

$$\delta\dot{r} \doteq \frac{dR}{\sqrt{2}} \sqrt{\left(\frac{v^2 - \dot{r}^2}{r^2}\right) + \omega^2 \sin^2\phi} \tag{27}$$

where ϕ is the angle between the satellite position vector $\vec{r}$ and the vector of the earth's rotational axis $\vec{\omega}$.

Equation 27 thus shows the relationship as mentioned at the beginning of this section. This equation gives as aimed at a rather simple relationship in order to obtain a "feel" for the situation and to estimate the error of $\dot{r}$ that is $\delta\dot{r}$.

Example: (1) Deep space mission (Reference 19)

$$d\dot{r} = 0.01 \text{ cm/s}$$

$$r \rightarrow \text{very large or } \left(\frac{v^2 - \dot{r}^2}{r^2}\right) \sim 0$$

$$\delta R = \frac{\sqrt{2} \cdot \delta \dot{r}}{\sin \phi \cdot \omega} = \frac{1.4 \times 10^{-2}}{0.63 \times 7.3 \times 10^{-5}}$$

$$= 300 \text{ cm} = 3 \text{ meters} .$$

(2) Near earth mission (Reference 8)

$$\delta \dot{r} = 3 \text{ cm/s} = 0.03 \text{ m/s}$$

$$v = 8 \text{ km/s}$$

$$\dot{r} = 6 \text{ km/s}$$

$$r = 500 \text{ km}$$

then $\alpha^2 \sin^2 \phi$ can be neglected in Equation 27 so that

$$\delta \dot{r} = \frac{\delta R}{\sqrt{2}} \frac{\sqrt{v^2 - \dot{r}^2}}{r} \qquad \text{or}$$

also

$$\delta R = \sqrt{2}\, \delta \dot{r} \frac{r}{\sqrt{v^2 - \dot{r}^2}}$$

$$= 1.4 \; 3 \; 10^{-2} \frac{500}{\sqrt{64 - 36}} \doteq 4 \text{ meters} .$$

Please note that considerable different range rate errors ($\delta \dot{r} = 0.01$ cm/s, 3 cm/s) were used based upon past and present experience for these particular missions.

Nevertheless, it shows that station location is important if one intends to make "full use" of the range rate information.

For a Radar System

Suppose that without measuring $\dot{r}$ as such (another criterion) the position error must be introduced in a radar system. In general, if the station position error $\delta \vec{R}$ is smaller than the radar error $\delta \vec{r}$, no improvements can be obtained any longer by the radar system. That is

$$\delta \vec{R} \doteq \delta \vec{r} \tag{28}$$

or

$$\delta\vec{R} = \sqrt{\delta r^2 + r^2\left(\delta\epsilon^2 + \delta\alpha^2\right)} \tag{29}$$

Using the same reason (neglecting δr^2) as for Equation 17, one obtains for the station error (square root of the number of the components)

$$\delta R \doteq \sqrt{2}\, r \left(\frac{\delta\measuredangle}{\sqrt{N}} + \delta\measuredangle_B\right) , \tag{30}$$

where $\delta\measuredangle = \delta\alpha = \delta\epsilon$, N the number of measurements used to "construct" one radar point (a vector, $\vec{r}(r, \alpha, \epsilon)$) and $\delta\measuredangle_B$ is the angular bias error.

Example: Let $N = 10$, $r = 300$ km, $\delta\measuredangle = 0.02$ mrad, $\delta\measuredangle_B = 0$, and 0.02 mrad. Then we obtain:

$$\delta R = 11 \text{ meters}$$

and

$$\delta R = 3.5 \text{ meters} ,$$

depending on the bias error $\delta\measuredangle_B$. These figures are believed to give a reasonable range of station accuracies.

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Greenbelt, Maryland, July 27, 1967
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